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Estimating double tuned mass dampers for structures under ground acceleration using a novel optimum criterion

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Abstract

The double tuned mass dampers (DTMD), consisting of one larger mass block (i.e. one larger tuned mass damper (TMD)) and one smaller mass block (i.e. one smaller TMD), have been proposed to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. The structure is represented by the mode-generalized system corresponding to the specific vibration mode that needs to be controlled. In light of the developed dynamic magnification factors (DMF) of the DTMD structure system, the criterion used for assessing the optimum parameters and effectiveness of the DTMD is selected as the minimization of the minimum values of the maximum DMF of the structure with the DTMD. With resorting to the maximum DMF of both the larger and smaller TMDs in the DTMD, the stroke of the DTMD is simultaneously investigated too. It is highlighted that a novel optimum objective function has been proposed in order to acquire high robust control system. Consequently, the two types of optimum goal functions (including the optimum goal function commonly used) have been applied for the optimum searching of the DTMD. The numerical results indicate that the DTMD designed in terms of the second type of optimum objective functions (i.e. the novel optimum objective function) practically provides the same effectiveness and robustness to the changes in the drift frequency ratio (DFR) as the multiple tuned mass dampers (MTMD) with the distributed natural frequencies with the total number of the TMD units equal to five and with equal total mass ratio. Likewise, the DTMD designed with resort to the second type of optimum objective functions can practically attain the same effectiveness as the TMD with equal total mass ratio. More importantly, in the robustness to the changes in the DFR, the DTMD is significantly better than the TMD, whereas in the robustness to the natural frequency tuning (NFT), measured by the frequency band width coefficient (FBWC), the DTMD is significantly better than the MTMD, thus manifesting that the DTMD is an advanced control device.

1. Introduction

In recent years, mitigating the dynamic responses of civil engineering structures to environmental loads such as earthquakes and wind loads has drawn the interest of many researchers. Many control devices, passive, semi-active, as well as active, have been developed. Among these available devices, the tuned mass damper (TMD) is one of the simplest and the most reliable control devices, which consists of a mass, a spring, and a

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viscous damper attached to the structure that is to be controlled. Its mechanism of attenuating undesirable vibrations of a structure is to transfer the vibration energy of the structure to the TMD and to dissipate the energy through the damping of the TMD. It is well known that however, the inherent limitations of a single TMD are the very narrow band of suppression frequency and the sensitivity to the fluctuation in tuning the frequency of the TMD to the controlled frequency of the structure, and the offset in the optimum damping of the TMD. The mistuning or off-optimum damping will reduce the effectiveness of the TMD significantly. Iwanami and Seto [1] proposed the dual tuned mass dampers (2TMD) and made research on the optimum design of 2TMD for harmonically forced oscillation of the structure. It was shown in their papers that 2TMD are more effective than a single TMD. However, the effectiveness was not significantly improved. Employing more than one TMD with different dynamic characteristics has then been proposed to further improve the effectiveness and robustness of the TMD. The multiple tuned mass dampers (MTMD) with the distributed natural frequencies were proposed by Xu and Igusa [2] and also studied by, for example, Yamaguchi and Harnpornchai [3], Abe and Fujino [4], Abe and Igusa [5], Kareem and Kline [6], Jangid [7,8], Joshi and Jangid [9], Bakre and Jangid [10], Kamiya et al. [11], Li [12], Li and Liu [13], Li and Li [14], Han and Li [15], Park and Reed [16], Gu et al. [17], Chen and Wu [18], Yau and Yang [19,20], Kwon and Park [21], Lin et al. [22], Wang and Lin [23], and Li and Qu [24]. The MTMD is shown to possess better effectiveness and higher robustness in mitigating the oscillations of structures with respect to a single TMD.

Recently, based on the various combinations available of the stiffness, mass, damping coefficient, and damping ratio in the MTMD, the five MTMD models have been presented by Li [25]. Through the implementation of both the minimization of the minimum values of the maximum displacement dynamic magnification factors (i.e. min.min.max.DDMF) and minimization of the minimum values of the maximum acceleration dynamic magnification factors (i.e. min.min.max.ADMF) of structures with the MTMD, it has been shown that the MTMD with the identical stiffness (i.e. $k_{T1} = k_{T2} = \cdots = k_{Tn} = k_T$) and damping coefficient (i.e. $c_{T1} = c_{T2} = \cdots = c_{Tn} = c_T$) but unequal mass (i.e. $m_{T1} \neq m_{T2} \neq \cdots \neq m_{Tn}$) and damping ratio (i.e. $\xi_{T1} \neq \xi_{T2} \neq \cdots \neq \xi_{Tn}$) provides better effectiveness and wider optimum frequency spacing (identical to higher robustness against the change or the estimation error in the structural controlled natural frequency) with respect to the rest of the MTMD models [25]. Likewise, the studies conducted by Li and Liu [26] have disclosed further trends of both the optimum parameters and effectiveness and further provided suggestion on selecting the total mass ratio and total number of the MTMD with the identical stiffness and damping coefficient but unequal mass and damping ratio. More recently, based on the uniform distribution of system parameters, instead of the uniform distribution of natural frequencies, the eight new MTMD models have been, for the first time, proposed in order to seek for the MTMD models without the near-zero optimum average damping ratio. Found are the six MTMD models without the near-zero optimum average damping ratio. The optimum MTMD with the identical damping coefficient (i.e. $c_{T1} = c_{T2} = \cdots = c_{Tn} = c_T$) and damping ratio (i.e. $\xi_{T1} = \xi_{T2} = \cdots = \xi_{Tn} = \xi_T$) but unequal stiffness (i.e. $k_{T1} \neq k_{T2} \neq \cdots \neq k_{Tn}$) and with the uniform distribution of masses is found able to render better effectiveness and wider optimum frequency spacing with respect to the rest of the MTMD models [27]. Likewise, it is interesting to find out that the two abovementioned MTMD models can approximately reach the same effectiveness and robustness [27]. The abovementioned review clearly shows that much progress has been extended in recent years in terms of the studies on the MTMD for the vibration control of structures.

However, to date, most researchers working on the MTMD system have assumed that the total number of the TMD units constituting the MTMD is an *odd number*, referred to as the *odd number* based MTMD, by targeting at the central natural frequency. The *arbitrary integer* based MTMD have been proposed by Li and Zhang [28] for the purpose of convenience in application of the MTMD by abandoning the central natural frequency hypothesis. Evidently, the idea of *arbitrary integer*, compared with *odd number*, should be more versatile in accommodating the requirements in practical situations. Likewise, the dual-layer multiple tuned mass dampers, referred to as the DL-MTMD, consisting of one larger TMD and several smaller TMDs with the total number of TMD units being the *arbitrary integer* and with the uniform distribution of natural frequencies have been further proposed by Li [29] to seek for the mass dampers with high effectiveness and robustness for the reduction of the undesirable vibrations of structures under the ground acceleration. The numerical results indicate that the DL-MTMD can render better effectiveness and higher robustness to the change in the natural frequency tuning (NFT), in comparison with the MTMD with the distributed natural frequencies with equal total mass ratio [29]. In fact, the DL-MTMD will degenerate into the double tuned mass dampers (DTMD) when the total number of the smaller TMD units in the DL-MTMD is set to be equal to unity. The investigations by Li [29] have manifested that the DL-MTMD has a little better effectiveness with respect to the DTMD; but they practically reach the same level of robustness to the change in the NFT. The DTMD consists of one larger mass block (i.e. larger TMD) and one smaller mass block (i.e. smaller TMD), thus implying that it is significantly simpler to manufacture the DTMD in comparison with the DL-MTMD. With a view to the engineering design and practical applications, it is imperative and of practical interest to carry on further investigations on the DTMD. Therefore, the main objective of this paper is focused on evaluating the performance of the DTMD (including assessing the stroke of both the larger and smaller TMDs in the DTMD) in order to demonstrate that the DTMD is an advanced control device in the mitigation of undesirable oscillations of structures under ground acceleration, using the two types of optimum objective functions, further designated as the first type of optimum goal functions commonly used, such as the study by Li and Qu [30] and second type of optimum goal functions (namely the novel optimum goal function proposed in the present paper).

2. Transfer functions (TFs) of the DTMDs structure system

In this paper, the DTMD are taken into account for the suppression of the specific vibration mode to be controlled of an MDOF structure. By use of the mode reduced-order method, the MDOF structure is modeled as an SDOF system, characterized by the mode-generalized stiffness (k_s), damping coefficient (c_s), and mass (m_s), respectively. Larger TMD (m_1) and smaller TMD (m_2) in the DTMD also are, respectively, modeled as an SDOF system. As a result, the total number of degrees-of-freedom of the DTMD structure system is equal to 3, as shown in Fig. 1. The following analysis to be carried out is based on this combined system. The equations of motion for the DTMD structure system subjected to the ground motion can be expressed as follows:

$$
\begin{aligned}
&m_s\ddot{x}_s + [c_s\dot{x}_s + c_1(\dot{x}_s - \dot{x}_1)] + [k_s x_s + k_1(x_s - x_1)] = -m_s\ddot{x}_g(t),\\
&m_1\ddot{x}_1 + [c_1(\dot{x}_1 - \dot{x}_s) + c_2(\dot{x}_1 - \dot{x}_2)] + [k_1(x_1 - x_s) + k_2(x_1 - x_2)] = -m_1\ddot{x}_g(t),\\
&m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = -m_2\ddot{x}_g(t).
\end{aligned}
\tag{1−3}
$$

Here, x_s is the displacement of the main structure with reference to the ground; x_1 represents the displacement of larger TMD in the DTMD with reference to the ground; x_2 denotes the displacement of smaller TMD in the DTMD with reference to the ground; m_1, k_1, and c_1 are, respectively, the mass, stiffness, and damping

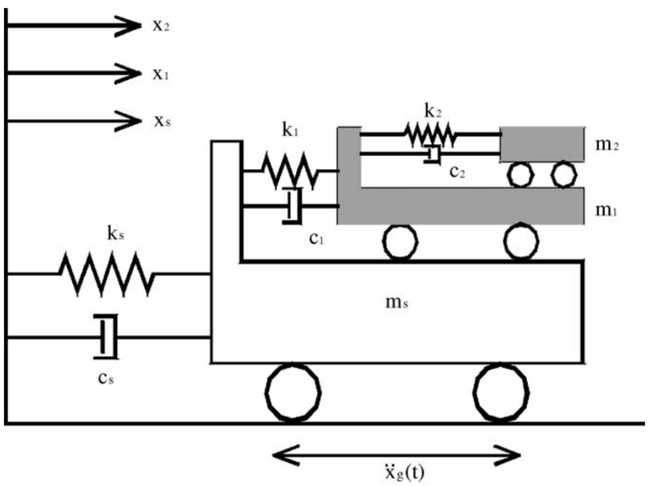

Fig. 1. Schematic diagram of the main structure with the double tuned mass dampers (TMDs) consisting of one larger TMD and one smaller TMD (referred to in the present paper as the DTMD) subjected to the ground acceleration.

coefficient of larger TMD in the DTMD; while m_2, k_2, and c_2 are, respectively, the mass, stiffness, and damping coefficient of smaller TMD in the DTMD; and $\ddot{x}_g(t)$ refers to the ground acceleration.

Transfer Eqs. (1–3) into the frequency domain form with resorting to the Laplace transform, namely $Z(s) = L[z(t)]$, which can be written, respectively, as follows:

$$\begin{aligned}
&(m_s X_s)s^2 + [c_s X_s + c_1(X_s - X_1)]s + [k_s X_s + k_1(X_s - X_1)] = -m_s \ddot{X}_g,\\
&(m_1 X_1)s^2 + [c_1(X_1 - X_s) + c_2(X_1 - X_2)]s + [k_1(X_1 - X_s) + k_2(X_1 - X_2)] = -m_1 \ddot{X}_g,\\
&(m_2 X_2)s^2 + [c_2(X_2 - X_1)]s + [k_2(X_2 - X_1)] = -m_2 \ddot{X}_g.
\end{aligned} \tag{4-6}$$

in which

$$\begin{aligned}
X_s &= X_s(s) = L[x_s],\\
X_1 &= X_1(s) = L[x_1],\\
X_2 &= X_2(s) = L[x_2],\\
\ddot{X}_g &= \ddot{X}_g(s) = L[\ddot{x}_g(t)].
\end{aligned} \tag{7-10}$$

Introduce the following parameters:

$$\begin{aligned}
&\mu_1 = \frac{m_1}{m_s};\ \ \mu_2 = \frac{m_2}{m_s};\ \ \lambda_1 = \frac{\omega_1}{\omega_0};\ \ \lambda_2 = \frac{\omega_2}{\omega_0};\\
&\zeta_s = \frac{c_s}{2m_s\omega_0};\ \ \zeta_1 = \frac{c_1}{2m_1\omega_1};\ \ \zeta_2 = \frac{c_2}{2m_2\omega_2};\\
&\omega_1 = \sqrt{k_1/m_1};\quad \omega_2 = \sqrt{k_2/m_2}
\end{aligned}$$

in which ω_0 is the design natural frequency of the main structure. Evidently, the design natural frequency of the main structure is always changing due to external excitations, and hence the natural frequency which deviates the design value is designated herein as ω_s. In view of this, the drift frequency ratio (DFR) bounded to the range from 0.5 to 1.5 is then introduced, which has the following form:

$$\gamma = \frac{\omega_s}{\omega_0}. \tag{11}$$

It is worth pointing here out that the different DFR, γ is taken into consideration in the present paper; and then for the purpose of convenience, we further introduce the notation: $\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_{n-1}, \gamma_n]$, referred to as the drift frequency ratio vector (DFRV). In fact, the DFR, γ may be set to be equal to any γ_i $(i = 1, 2, \ldots, n)$.

Eqs. (4–6) may then be rewritten as

$$\begin{aligned}
&X_s s^2 + [2\omega_0\zeta_s X_s + 2\lambda_1\omega_0\zeta_1\mu_1(X_s - X_1)]s + [\gamma^2\omega_0^2 X_s + \mu_1\lambda_1^2\omega_0^2(X_s - X_1)] = -\ddot{X}_g,\\
&(\mu_1 X_1)s^2 + [2\lambda_1\omega_0\zeta_1\mu_1(X_1 - X_s) + 2\lambda_2\omega_0\zeta_2\mu_2(X_1 - X_2)]s\\
&\quad + [\mu_1\lambda_1^2\omega_0^2(X_1 - X_s) + \mu_2\lambda_2^2\omega_0^2(X_1 - X_2)] = -\mu_1\ddot{X}_g,\\
&(\mu_2 X_2)s^2 + [2\lambda_2\omega_0\zeta_2\mu_2(X_2 - X_1)]s + [\mu_2\lambda_2^2\omega_0^2(X_2 - X_1)] = -\mu_2\ddot{X}_g.
\end{aligned} \tag{12-14}$$

Eqs. (12–14) can be rewritten into the matrix equation form as follows:

$$\begin{bmatrix} A(s) & B(s) & 0 \\ B(s) & F(s) & G(s) \\ 0 & G(s) & I(s) \end{bmatrix} \begin{bmatrix} X_s \\ X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -\mu_1 \\ -\mu_2 \end{bmatrix} \ddot{X}_g. \tag{15}$$

Solving Eq. (15) then results in the TFs of the DTMD structure system, such that

$$\mathrm{TF}_s(s) = \frac{X_s}{\ddot{X}_g} = \frac{\mu_1[B(s)I(s)] - \mu_2[B(s)G(s)] + [G(s)G(s)] - [F(s)I(s)]}{[A(s)F(s)I(s)] - [A(s)G(s)G(s)] - [B(s)B(s)I(s)]},$$
$$\mathrm{TF}_{m1}(s) = \frac{X_1}{\ddot{X}_g} = \frac{-\mu_1[A(s)I(s)] + \mu_2[A(s)G(s)] + [B(s)I(s)]}{[A(s)F(s)I(s)] - [A(s)G(s)G(s)] - [B(s)B(s)I(s)]},$$
$$\mathrm{TF}_{m2}(s) = \frac{X_2}{\ddot{X}_g} = \frac{\mu_1[A(s)G(s)] - \mu_2[A(s)F(s)] - [B(s)G(s)] + \mu_2[B(s)B(s)]}{[A(s)F(s)I(s)] - [A(s)G(s)G(s)] - [B(s)B(s)I(s)]}. \quad (16-18)$$

In which $\mathrm{TF}_s(s)$ denotes the TF of the main structure with the DTMD; $\mathrm{TF}_{m1}(s)$ is the TF of the larger TMD in the DTMD; and $\mathrm{TF}_{m2}(s)$ refers to the TF of the smaller TMD in the DTMD. $A_s(s)$, $B(s)$, $F(s)$, $G(s)$, and $I(s)$ may be calculated, respectively, as follows:

$$A(s) = s^2 + (2\omega_0\zeta_s + 2\lambda_1\omega_0\zeta_1\mu_1)s + \gamma^2\omega_0^2 + \mu_1\lambda_1^2\omega_0^2,$$
$$B(s) = (-2\lambda_1\omega_0\zeta_1\mu_1)s - \mu_1\lambda_1^2\omega_0^2,$$
$$F(s) = \mu_1 s^2 + (2\lambda_1\omega_0\zeta_1\mu_1 + 2\lambda_2\omega_0\zeta_2\mu_2)s + \mu_1\lambda_1^2\omega_0^2 + \mu_2\lambda_2^2\omega_0^2,$$
$$G(s) = (-2\lambda_2\omega_0\zeta_2\mu_2)s - \mu_2\lambda_2^2\omega_0^2,$$
$$I(s) = \mu_2 s^2 + (2\lambda_2\omega_0\zeta_2\mu_2)s + \mu_2\lambda_2^2\omega_0^2.$$

3. Dynamic magnification factors (DMF) of the DTMDs structure system

Express the DMF of the DTMD structure system through setting $s = i\omega$ as follows:

$$\mathrm{DMF}(\omega_s, \omega) = \mathrm{DMF}(\gamma\omega_0, \omega) = \left|\omega_s^2[\mathrm{TF}_s(\mathrm{i}\omega)]\right| = (\gamma^2\omega_0^2)\left|[\mathrm{TF}_s(\mathrm{i}\omega)]\right|,$$
$$\mathrm{DMF}_{m1}(\omega_s, \omega) = \mathrm{DMF}_{m1}(\gamma\omega_0, \omega) = \left|\omega_s^2[\mathrm{TF}_{m1}(\mathrm{i}\omega)]\right| = (\gamma^2\omega_0^2)\left|[\mathrm{TF}_{m1}(\mathrm{i}\omega)]\right|,$$
$$\mathrm{DMF}_{m2}(\omega_s, \omega) = \mathrm{DMF}_{m2}(\gamma\omega_0, \omega) = \left|\omega_s^2[\mathrm{TF}_{m2}(\mathrm{i}\omega)]\right| = (\gamma^2\omega_0^2)\left|[\mathrm{TF}_{m2}(\mathrm{i}\omega)]\right|. \quad (19-21)$$

In which $\mathrm{DMF}(\omega_s, \omega) = \mathrm{DMF}(\gamma\omega_0, \omega)$ denotes the DMF of the main structure with the DTMD; $\mathrm{DMF}_{m1}(\omega_s, \omega) = \mathrm{DMF}_{m1}(\gamma\omega_0, \omega)$ represents the DMF of the larger TMD in the DTMD; and $\mathrm{DMF}_{m2}(\omega_s, \omega) = \mathrm{DMF}_{m2}(\gamma\omega_0, \omega)$ refers to the DMF of the smaller TMD in the DTMD.

4. Two types of optimum objective functions—first type of optimum objective functions commonly used and second type of optimum objective functions (namely the novel optimum objective function)

Introduce the following two non-dimensional parameters:

Total mass ratio of the total mass of the DTMD to the mode-generalized mass of structures:

$$\mu = \mu_1 + \mu_2, \quad (22a)$$

mass ratio of the smaller to larger TMD mass:

$$\mu_H = \frac{m_2}{m_1} = \frac{\mu_2}{\mu_1}. \quad (22b)$$

In order to design the DTMD with high robustness, the several natural frequencies of the main structure with the DTMD will be taken into consideration, which have the following form:

$$\Gamma\omega_0 = [\gamma_1, \gamma_2, \ldots, \gamma_{n-1}, \gamma_n]\omega_0,$$
$$\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_{n-1}, \gamma_n], \quad (23)$$

where each γ_i is set within the range from 0.5 to 1.5.

Then, a novel optimum objective function, namely the second type of optimum goal functions, of the DTMD structure system is given by

$$R = \min . \sum_{i=1}^{n} \left[\alpha_i \times [\max . \mathrm{DMF}(\gamma_i \omega_0, \omega)] \right] = \min . [\alpha\beta] \tag{24}$$

in which

$$\alpha = [\alpha_1, \alpha_2, \ldots \alpha_{n-1}, \alpha_n],$$
$$\beta = \begin{bmatrix} [\max . \mathrm{DMF}(\gamma_1 \omega_0, \omega)] \\ [\max . \mathrm{DMF}(\gamma_2 \omega_0, \omega)] \\ \vdots \\ [\max . \mathrm{DMF}(\gamma_{n-1} \omega_0, \omega)] \\ [\max . \mathrm{DMF}(\gamma_n \omega_0, \omega)] \end{bmatrix}, \tag{25, 26}$$

where α_i is a weighting factor. Evidently, larger weighting factor α_i corresponds to attaching much importance to the maximum DMF of the main structure with the DTMD at $\gamma_i\omega_0$, namely $[\max . \mathrm{DMF}(\gamma_i \omega_0, \omega)]$. Besides, the external excitation frequency ω is set within the range from $0.4\gamma_i\omega_0$ to $4.4\gamma_i\omega_0$. When α and β simultaneously meet with Eqs. (32,33), Eq. (24) will degenerate into the first type of optimum objective functions commonly used.

The implementation of Eq. (24) will then yield the optimum values of λ_1, λ_2, ζ_1, ζ_2, and μ_H. It is worth pointing here out that when using Eq. (24), there is a need for meeting with

$$\lambda_1, \lambda_2 \geqslant \varepsilon; \quad \mu_H \geqslant \varepsilon;$$
$$\zeta_2 \in [0, 1); \quad \zeta_1 \in [0, 1) \tag{27–29}$$

in which $\varepsilon > 0$ is a non-negative scalar, which approaches zero. Introduction of this number is to avoid the occurrence of singular stiffness matrix and/or singular mass matrix in the numerical optimality.

Subsequently, the DTMD stroke can be simultaneously evaluated through assessing the maximum DMF of both the larger and smaller TMDs in the DTMD using the optimum parameters of the DTMD obtained in light of the optimum criterion in Eq. (24), which has the following forms:

$$R_{m1}(\omega_s, \omega) = R_{m1}(\gamma_i \omega_0, \omega) = \max . [\mathrm{DMF}_{m1}(\gamma_i \omega_0, \omega)],$$
$$R_{m2}(\omega_s, \omega) = R_{m2}(\gamma_i \omega_0, \omega) = \max . [\mathrm{DMF}_{m2}(\gamma_i \omega_0, \omega)]. \tag{30, 31}$$

5. Numerical studies

5.1. Designing the DTMD with the first type of optimum objective functions commonly used

In the case where the structural controlled natural frequency keeps unchanged, namely the DFR is set to be equal to unity, the parameters in the optimum criterion (Eq. (24)) can be set as follows:

$$\alpha = [0 \quad \cdots 0 \quad 1 \quad 0 \quad \cdots \quad 0],$$
$$\Gamma = [1 \quad \cdots \quad 1 \quad 1 \quad 1 \quad \cdots \quad 1]. \tag{32, 33}$$

It is worth mentioning herein that the optimum goal function (Eq. (24)), which meets with Eqs. (32,33), is referred to as the first type of optimum goal functions, which is commonly used, such as Ref. [30].

With resorting to the optimum criterion (Eq. (24)), we obtain the optimum parameters of the DTMD with the total mass ratio equal to 0.03 as follows:

$$\begin{aligned} \lambda_1 &= 1.0086, \\ \lambda_2 &= 0.9321, \\ \zeta_1 &= 0.0000, \\ \zeta_2 &= 0.1987, \\ \mu_H &= 0.0830. \end{aligned} \tag{34}$$

Fig. 2 shows the frequency response function (FRF) of the main structure, respectively, with the DTMD, TMD, and MTMD with the total mass ratio equal to 0.03, in the case of DFRV, $\Gamma = [0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2]$, using the first type of optimum goal functions commonly used, namely Eq. (24) but simultaneously meeting with Eqs. (32,33). It is worth pointing herein out that the MTMD in Fig. 2 directly uses that in Ref. [12], with the total number of the TMD units equal to five. It is seen clearly from Fig. 2 that both the effectiveness and robustness to the changes in the DFR for the DTMD are higher than those for both the MTMD and TMD.

Table 1 gives the Max.DMF of the main structure with the DTMD, MTMD, and TMD designed using the first type of optimum objective functions commonly used; whereas Table 2 lists the Max.DMF of every TMD unit in both the DTMD and MTMD, as well as the TMD designed using the first type of optimum objective functions commonly used. Drawn from Tables 1 and 2, what worth noting is that the optimum damping ratio of the larger TMD equals zero, thus no need of the dampers between the larger TMD in the DTMD and the structure. Notwithstanding this, the maximum DMF of the larger TMD, used for estimating the stroke, always remains at a lower level. For different DFR values, the max.DMF of both the larger and smaller TMDs in the DTMD, every TMD unit, ordered increasingly in light of the natural frequencies, in the MTMD, as well as the TMD is listed in Tables 1 and 2 for the purpose of comparisons. From Tables 1 and 2, the

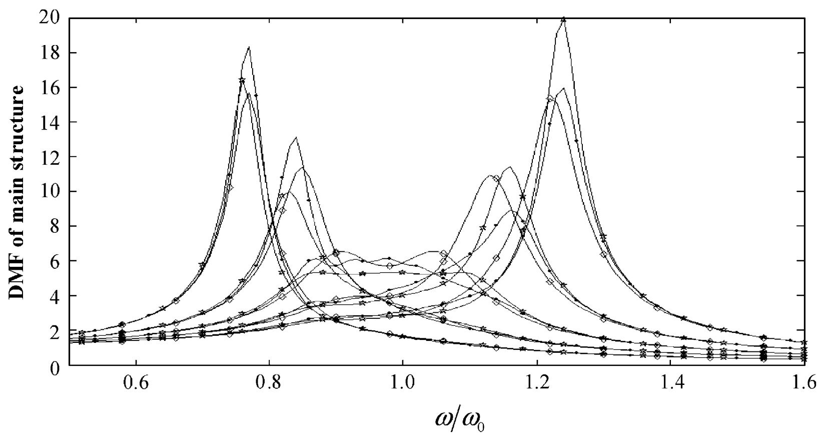

Fig. 2. Frequency response function (FRF) of the main structure respectively with the DTMD, TMD, and MTMD in the case of DFRV, $\Gamma = [0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2]$ using the first type of optimum goal functions commonly used: DTMD (—☆—); TMD (—◇—); MTMD (—•—).

Table 1
The Max.DMF of the main structure respectively with the DTMD, TMD, and MTMD designed in terms of the first type of optimum objective functions commonly used

DFR, γ	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Max.DMF of the main structure with the DTMD	15.363	17.564	16.409	10.004	5.347	11.425	19.888
Max.DMF of the main structure with the TMD	14.884	16.630	15.691	11.400	6.546	10.926	15.343
Max.DMF of the main structure with the MTMD	15.344	17.927	18.307	13.140	6.131	8.902	15.938

Table 2
The Max.DMF of every tuned mass damper unit in the DTMD and MTMD, as well as the TMD designed in terms of the first type of optimum objective functions commonly used

DFR, γ	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Larger tuned mass damper (m_1) in the DTMD	24.7	36.7	48.9	45.2	32.3	39.2	44.5
Smaller tuned mass damper (m_2) in the DTMD	39.3	69.7	110.8	117.9	89.4	70.3	63.4
m in the TMD	23.7	32.8	41.3	42.7	34.0	25.4	23.9
First tuned mass damper (m_1) in the MTMD	27.4	43.4	72.3	106.5	84.4	62.5	55.3
Second tuned mass damper (m_2) in the MTMD	25.1	37.2	54.0	61.2	75.0	59.1	50.5
Third tuned mass damper (m_3) in the MTMD	23.4	33.3	44.5	44.2	65.8	59.7	48.4
Fourth tuned mass damper (m_4) in the MTMD	22.2	30.5	38.7	35.6	51.3	62.9	48.3
Fifth tuned mass damper (m_5) in the MTMD	21.2	28.5	34.8	30.5	37.8	72.1	64.8

following main conclusion can be obtained:

(1) The stroke of the larger TMD is significantly smaller than that of the smaller TMD; likewise the former is close to the TMD in terms of the stroke, whereas the stroke of the latter approaches the maximum stroke of the MTMD. Considering that the smaller TMD possesses higher optimum damping ratio, the vibration energy in the DTMD is mainly dissipated through the relative motion between both the larger and smaller TMDs. The large stroke of the smaller TMD will not pose much technological trouble in practical applications. The reason for this is attributed to its small mass, which can be explicitly demonstrated by the following expression:

$$\mu_2 = \frac{m_2}{m_s} = \mu\left[\frac{\mu_H}{1+\mu_H}\right] \approx \mu\mu_H. \tag{35}$$

(2) In the case of DFRV, $\Gamma = [0.7 \quad 0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2]$, namely when the DFR changes within the large range from 0.7 to 1.2, the smaller TMD always maintains larger stroke, which means that the DTMD can always render the capacity of vibration energy dissipation on a desirable level.
(3) For the case of great changes in the DFR, the TMD units, whose natural frequencies are away from the controlled natural frequency of the structure, possess lower stroke. For instance, when the DFR changes within the range from 0.6 to 0.9, the stroke of the fifth TMD unit with the natural frequency approximately equal to $1.114\omega_0$ is smaller than the rest of same column in Table 2, thus, in practical terms, playing a negligible role in reducing the vibration of structures and then indicating that the effective tuning mass of the MTMD will decrease due to dispersion of the tuning frequency band of the MTMD.

Fig. 3. presents the max.DMF of the main structure with the DTMD, TMD, and MTMD, respectively, with respect to the DFR, using the first type of optimum objective functions commonly used. It is seen that near DFR, $\gamma = 1$, the robustness of the DTMD to the changes in the DFR is very close to that of the MTMD, but both higher than the TMD in terms of the robustness. It is also shown that the DTMD renders higher effectiveness than both the TMD and MTMD. However, when the DFR keeps increasing from $\gamma = 1$, the DTMD will be worse than both the TMD and MTMD in terms of the effectiveness; when the DFR goes decreasing from $\gamma = 0.8$, the DTMD in terms of the effectiveness will be worse than the TMD but better than the MTMD. Therefore, the first type of optimum objective functions commonly used cannot render the DTMD with high robustness to large changes in the DFR.

It is importantly pointed herein out that when meeting with any one of the following five requirements:

$$\begin{aligned} &\zeta_1 \geqslant 0.000,\\ &\zeta_1 \geqslant 0.001,\\ &\zeta_1 \geqslant 0.005,\\ &\zeta_1 \geqslant 0.010,\\ &\zeta_1 \geqslant 0.050. \end{aligned} \tag{36}$$

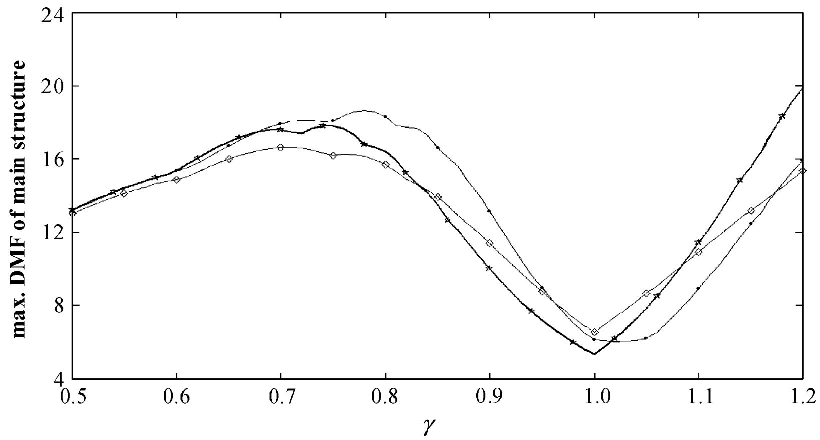

Fig. 3. Employing the first type of optimum objective functions commonly used, max.DMF of the main structure respectively with the DTMD, TMD, and MTMD with respect to the drift frequency ratio (DFR): DTMD (—☆—); TMD (—◇—); MTMD (—•—).

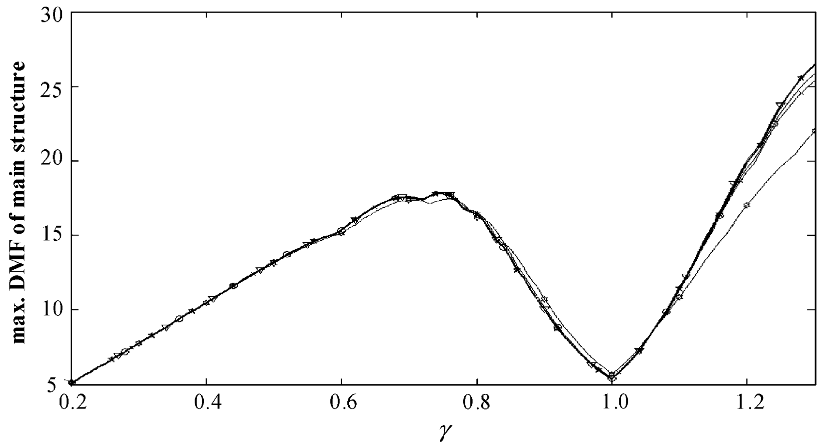

Fig. 4. Employing the first type of optimum objective functions commonly used, max.DMF of the main structure with the DTMD with different lower bounds of the damping ratio of the larger tuned mass damper with respect to the drift frequency ratio (DFR): $\zeta_1 \geqslant 0.0$ (—★—); $\zeta_1 \geqslant 0.001$ (—▽—); $\zeta_1 \geqslant 0.005$ (—○—); $\zeta_1 \geqslant 0.01$ (—×—); $\zeta_1 \geqslant 0.05$ (—☆—).

The implementation of the optimum criterion (Eq. (24)) always yields lower bounds of the damping ratio (ζ_1) of the larger TMD.

Employing the first type of optimum objective functions commonly used and considering different lower bounds of the damping ratio of the larger TMD, Figs. 4–6, respectively, show the max.DMF of the main structure with the DTMD, the larger TMD, and smaller TMD with respect to the DFR. As can be seen, the optimum damping ratio of the larger TMD is always equal to the lower bounds of the damping ratio, implying that the DTMD behaves quite insensitively to the changes in the damping ratio of the larger TMD, within the range from 0.0 to 0.001 or from 0.005 to 0.01. However, the max.DMF of both the larger and smaller TMDs fluctuates significantly with the lower bounds of the damping ratio rising to a higher level, such as 0.05.

5.2. Designing the DTMD with the second type of optimum objective functions (namely the novel optimum objective function)

In order to acquire higher robustness against large changes in the DFR, the importance should be attached to the structural DMF at different DFR; but much importance should be yet attached to the contribution of the DMF at the structural controlled natural frequency to the performance index (Eq. (24)).

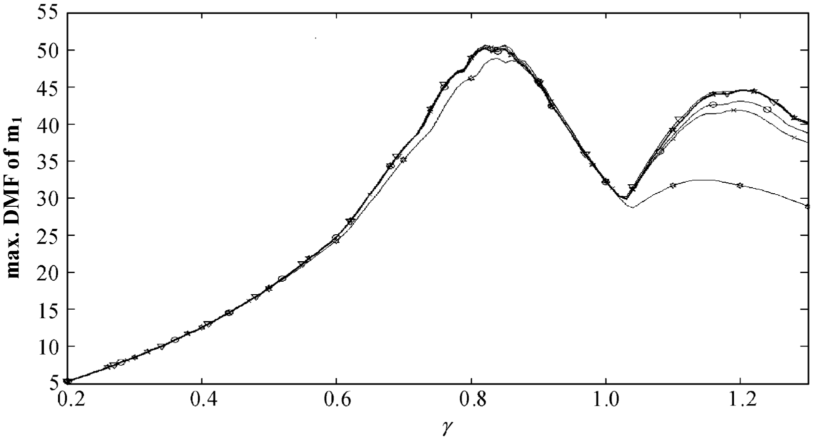

Fig. 5. Employing the first type of optimum objective functions commonly used, max.DMF of the larger tuned mass damper (m_1) in the DTMD with different lower bounds of the damping ratio of the larger tuned mass damper with respect to the drift frequency ratio (DFR): $\zeta_1 \geqslant 0.0$ (—★—); $\zeta_1 \geqslant 0.001$ (—▽—); $\zeta_1 \geqslant 0.005$ (—○—); $\zeta_1 \geqslant 0.01$ (—×—); $\zeta_1 \geqslant 0.05$ (—✡—).

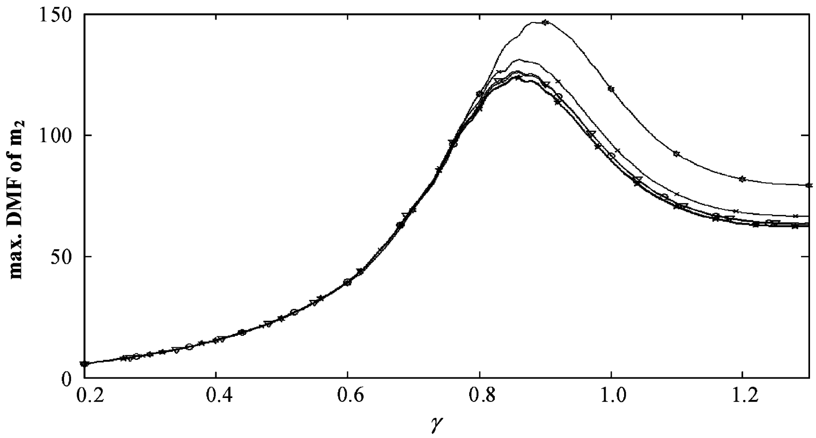

Fig. 6. Employing the first type of optimum objective functions commonly used, max.DMF of the smaller tuned mass damper (m_2) in the DTMD with different lower bounds of the damping ratio of the larger tuned mass damper with respect to the drift frequency ratio (DFR): $\zeta_1 \geqslant 0.0$ (—★—); $\zeta_1 \geqslant 0.001$ (—▽—); $\zeta_1 \geqslant 0.005$ (—○—); $\zeta_1 \geqslant 0.01$ (—×—); $\zeta_1 \geqslant 0.05$ (—✡—).

Letting

$$\begin{aligned} \alpha &= [1 \quad 1 \quad 5 \quad 1 \quad 1], \\ \Gamma &= [0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2], \end{aligned} \tag{37, 38}$$

the optimal parameters of the DTMD with resorting to the optimum criterion (Eq. (24)) as follows:

$$\begin{aligned} \lambda_1 &= 1.1048, \\ \lambda_2 &= 0.9634, \\ \zeta_1 &= 0.0000, \\ \zeta_2 &= 0.3514, \\ \mu_H &= 0.2010. \end{aligned} \tag{39}$$

Fig. 7 shows the FRF of the main structure with the DTMD, TMD, and MTMD, respectively, in the case of DFRV, $\Gamma = [0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2]$, using the optimum criterion (Eq. (24) under the condition of

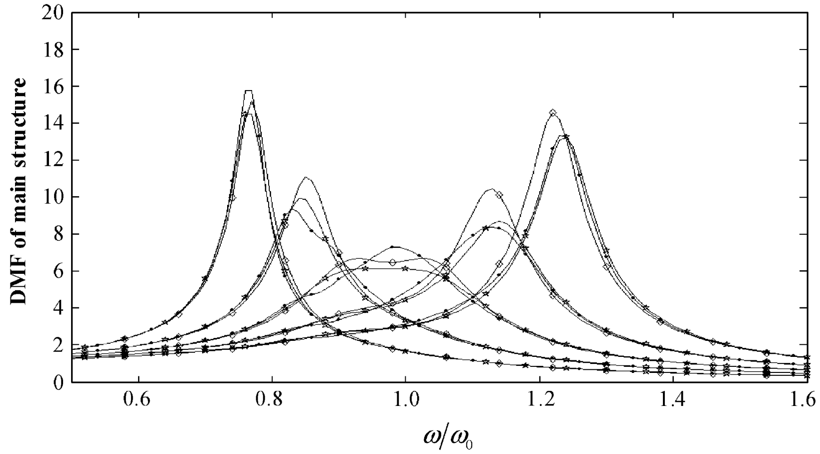

Fig. 7. Frequency response function (FRF) of the main structure respectively with the DTMD, TMD, and MTMD in the case of DFRV, $\Gamma = [0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2]$ using the second type of optimum goal functions (namely the novel optimum goal function): DTMD (—☆—); TMD (—◇—); MTMD (—•—).

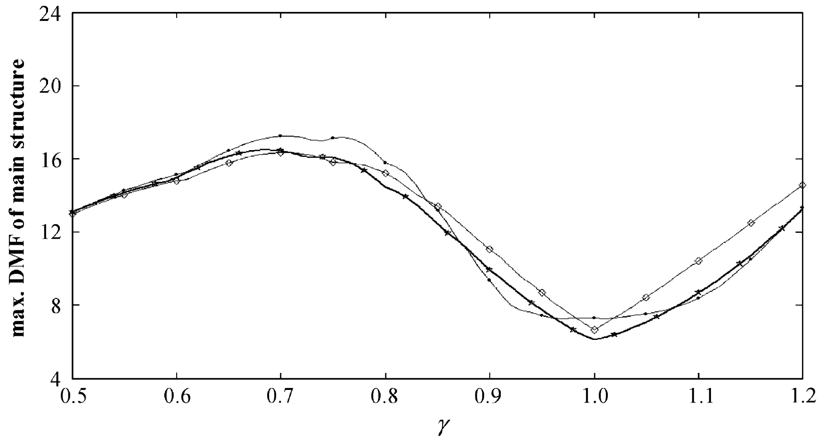

Fig. 8. Employing the second type of optimum objective functions (namely the novel optimum goal function), max.DMF of the main structure respectively with the DTMD, TMD, and MTMD with respect to the drift frequency ratio (DFR): DTMD (—☆—); TMD (—◇—); MTMD (—•—).

Eqs. (37,38)), referred herein to as the second type of optimum goal functions. It is seen from Fig. 7 that the DTMD designed using the second type of optimum objective functions may further enhance the robustness to the changes in the DFR. Fig. 8 presents the max.DMF of the main structure with the DTMD, TMD, and MTMD, respectively, with respect to the DFR using the second type of optimum objective functions. It is seen from Fig. 8 that near the structural controlled natural frequency, the max.DMF of the main structure with the MTMD possesses larger value. For instance, at $\gamma = 1$, the max.DMF of the main structure with the MTMD equals 6.131 using the first type of optimum objective functions; whereas the max.DMF of the main structure with the MTMD equals 7.248 using the second type of optimum objective functions. A possible explanation of such a phenomenon for the MTMD is that in a certain frequency band which could be tuned by the DTMD, MTMD, and TMD, the MTMD in terms of the effective tuning mass is smaller than both the DTMD and TMD due to using the discretized TMD units to control different natural frequencies distributed near the structural controlled natural frequency, thus rendering relatively lower level of vibration suppression. However, when large changes in the DFR happen, the effectiveness of both the DTMD and TMD decreases rapidly. Notwithstanding large changes in the DFR, the effectiveness of the MTMD practically keeps unchanged because its wide tunable frequency band prevents the effectiveness from the rapid drop. Therefore, it can be accounted that the MTMD is the best among three devices in the narrow natural frequency band width with the center at $\gamma = 1.0$ (about within the ranges from $\gamma = 0.9$ to 1.0 and from $\gamma = 1.0$ to 1.1). But at $\gamma = 1.0$, the DTMD can provide the highest effectiveness. However, when the DFR varies within the range from 0.6 to 0.85, the effectiveness of the MTMD drops rapidly due to smaller effective tuning mass than that of both the DTMD and TMD. What is more important is that the DTMD practically attains the same effectiveness and robustness as the MTMD with the total number of the TMD units equal to five (see Fig. 9) via further modifying the weighting factors in the second type of optimum objective function as follows:

$$\alpha = [2 \quad 2 \quad 6 \quad 3 \quad 3 \quad 13 \quad 5 \quad 4 \quad 4 \quad 3 \quad 3],$$
$$\Gamma = [0.8 \quad 0.85 \quad 0.9 \quad 0.9125 \quad 0.95 \quad 1.0 \quad 1.025 \quad 1.05 \quad 1.1 \quad 1.15 \quad 1.2].$$

Table 3 presents the Max.DMF of the main structure respectively with the DTMD, MTMD, and TMD designed using the second type of objective functions; while Table 4 renders the Max.DMF of each TMD unit in the DTMD and MTMD, as well as the TMD designed using the second type of objective functions. It is once again indicated in Tables 3 and 4 that the effective tuning mass is excessively impaired when large changes in the DFR happen. For example, when the DFR, $\gamma = 0.6, 0.7, 0.8, 0.9, 1.0$, the limited stroke of the fifth TMD unit in the MTMD means that it plays a negligible role in vibration reduction in practical terms. The level of vibration reduction rendered by every TMD unit, except for the fifth TMD unit, is not that satisfactory with the DFR above unity. The TMD provides lower level of vibration suppression due to smaller stroke at the DFR, $\gamma = 1.1, 1.2$. However, the stroke of the smaller TMD in the DTMD is always relatively large, which may account for its good effectiveness and robustness.

Table 5 shows the Max.DMF of both the larger and smaller TMDs in the DTMD as well as the TMD in the case of further modifying weighting factors in the second type of goal functions. Notwithstanding this further modification, the stroke of both the larger and smaller TMDs in the DTMD as well as the TMD does not yield

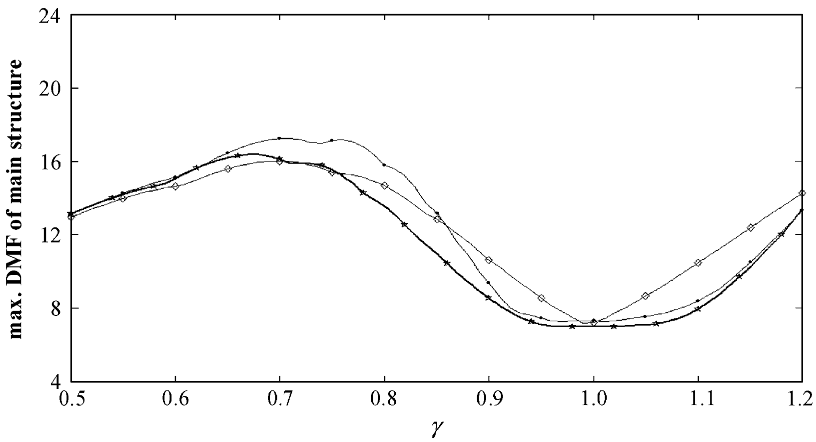

Fig. 9. Employing the second type of optimum objective functions (namely the novel optimum goal function) but further modifying the weighing factors in the optimum goal functions of both the DTMD and TMD, max.DMF of the main structure respectively with the DTMD, TMD, and MTMD with respect to the drift frequency ratio (DFR): DTMD (—☆—); TMD (—◇—); MTMD (—•—).

Table 3
The Max.DMF of the main structure respectively with the DTMD, TMD, and MTMD designed in terms of the second type of optimum objective functions (namely the novel optimum objective function)

DFR, γ	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Max.DMF of the main structure with the DTMD	14.964	16.435	14.468	9.938	6.112	8.672	13.241
Max.DMF of the main structure with the TMD	14.779	16.333	15.213	11.078	6.668	10.437	14.569
Max.DMF of the main structure with the MTMD	15.127	17.242	15.756	9.351	7.285	8.381	13.305

Table 4
The Max.DMF of every tuned mass damper unit in the DTMD and MTMD, as well as the TMD designed in terms of the second type of optimum objective functions (namely the novel optimum objective function)

DFR, γ	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Larger tuned mass damper (m_1) in the DTMD	23.3	32.4	38.8	37.1	29.2	28.2	32.8
Smaller tuned mass damper (m_2) in the DTMD	33.5	51.5	66.9	66.6	52.3	40.1	38.8
m in the TMD	23.4	31.8	38.7	38.6	30.6	24.2	22.9
First tuned mass damper (m_1) in the MTMD	28.4	45.4	72.1	79.8	51.1	40.1	37.1
Second tuned mass damper (m_2) in the MTMD	25.1	36.7	48.1	49.3	55.9	39.8	34.2
Third tuned mass damper (m_3) in the MTMD	23.0	31.8	37.7	30.2	55.2	44.4	33.7
Fourth tuned mass damper (m_4) in the MTMD	21.6	28.7	32.0	23.7	40.4	53.7	38.9
Fifth tuned mass damper (m_5) in the MTMD	20.5	26.6	28.5	20.1	27.6	54.6	60.6

Table 5
The Max.DMF of both the larger and smaller tuned mass dampers in the DTMD, as well as the TMD in the case of modifying weighting factors in the second type of optimum goal functions (namely the novel optimum objective function)

DFR, γ	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Larger tuned mass damper (m_1) in the DTMD	23.9	33.1	37.9	32.2	27.1	29.5	36.6
Smaller tuned mass damper (m_2) in the DTMD	37.0	59.4	76.7	69.7	53.4	43.4	40.3
m in the TMD	23.2	31.0	36.7	35.7	28.0	23.2	21.9

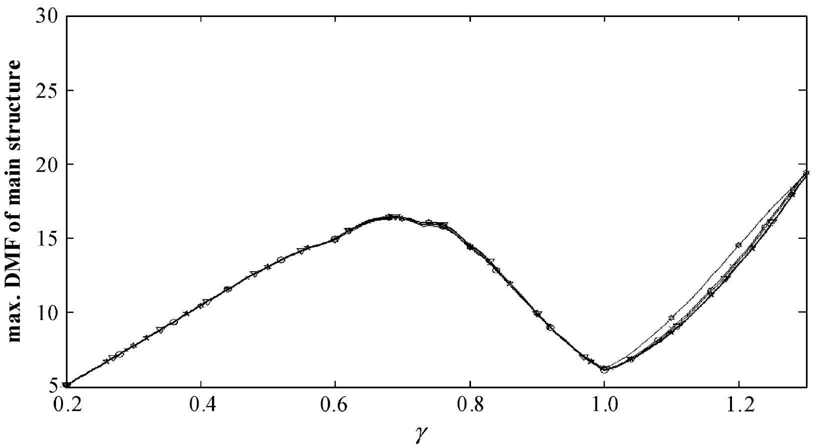

Fig. 10. Employing the second type of optimum objective functions (namely the novel optimum goal function), max.DMF of the main structure with the DTMD with different lower bounds of the damping ratio of the larger tuned mass damper with respect to the drift frequency ratio (DFR): $\zeta_1 \geqslant 0.0$ (—★—); $\zeta_1 \geqslant 0.001$ (—▽—); $\zeta_1 \geqslant 0.005$ (—○—); $\zeta_1 \geqslant 0.01$ (—×—); $\zeta_1 \geqslant 0.05$ (—✡—).

significant change. Here, it is once again confirmed that the optimum damping ratio of the large TMD in the DTMD yet equals zero though on the basis of the second type of optimum goal functions (namely the novel optimum goal function).

Employing the second type of optimum objective functions, Figs. 10–12, respectively, present the max.DMF of the main structure with the DTMD, the larger and smaller TMDs in the DTMD with different lower bounds of the damping ratio, with respect to the DFR. Similarly, the implementation of the optimality meets with any one of the five requirements (Eq. (36)). Figs. 10–12 once again manifests that though using the second type of optimum goal functions, the optimum damping ratio of the larger TMD yet equals the lower bounds of the damping ratio, indicating that the DTMD behaves quite insensitively to the changes in the damping ratio of the larger TMD, within the range from 0.0 to 0.001 or from 0.005 to 0.01. However, the max.DMF of

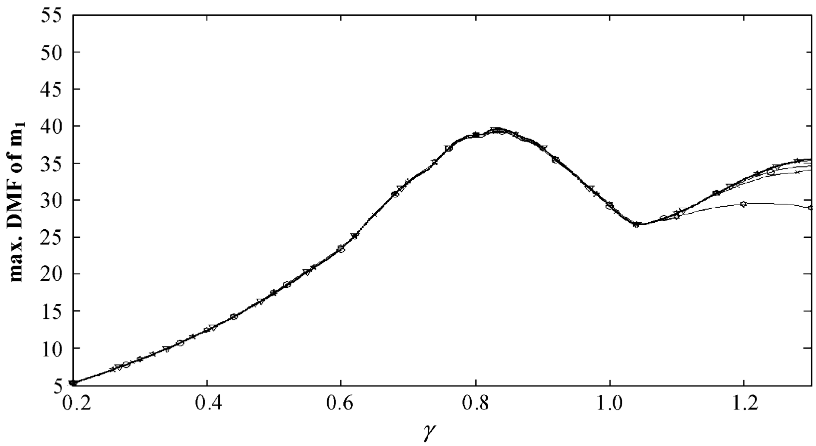

Fig. 11. Employing the second type of optimum objective functions (namely the novel optimum goal function), max.DMF of the larger tuned mass damper (m_1) in the DTMD with different lower bounds of the damping ratio of the larger tuned mass damper with respect to the drift frequency ratio (DFR): $\zeta_1 \geqslant 0.0$ (—★—); $\zeta_1 \geqslant 0.001$ (—▽—); $\zeta_1 \geqslant 0.005$ (—○—); $\zeta_1 \geqslant 0.01$ (—×—); $\zeta_1 \geqslant 0.05$ (—✡—).

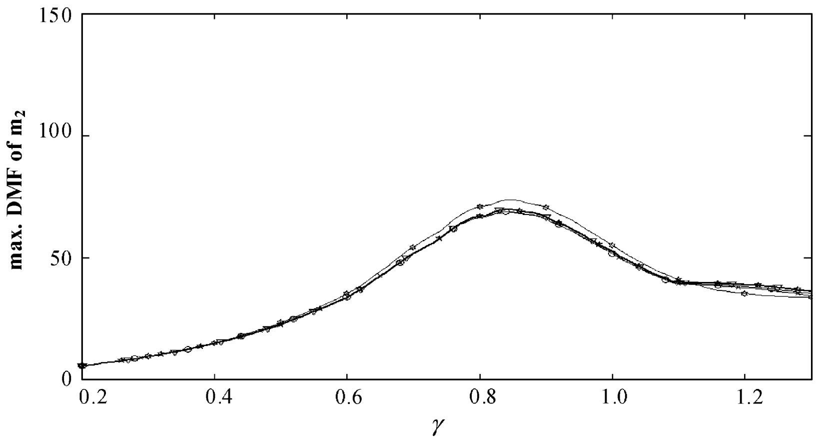

Fig. 12. Employing the second type of optimum objective functions (namely the novel optimum goal function), max.DMF of the smaller tuned mass damper (m_2) in the DTMD with different lower bounds of the damping ratio of the larger tuned mass damper with respect to the drift frequency ratio (DFR): $\zeta_1 \geqslant 0.0$ (—★—); $\zeta_1 \geqslant 0.001$ (—▽—); $\zeta_1 \geqslant 0.005$ (—○—); $\zeta_1 \geqslant 0.01$ (—×—); $\zeta_1 \geqslant 0.05$ (—✡—).

both the larger and smaller TMDs fluctuates significantly with the lower bounds of the damping ratio rising to a higher level, such as 0.05.

5.3. Comparison between two types of optimum objective functions

Table 6 lists the minimum max.DMF (i.e. min.max.DMF) of the structure with the DTMD, MTMD, and TMD using the first type of optimum objective functions commonly used and second type of optimum objective functions (namely the novel optimum objective function) as well as modifying the weighting factors in the second types of optimum objective functions for both the DTMD and TMD, when the DFR is equal to unity. From Table 6 (see also Fig. 13), the following conclusions are obtained:

(1) The DTMD can provide higher effectiveness with respect to the MTMD and TMD designed using the first type of optimum objective functions. Likewise, the DTMD is very close to the MTMD in terms of the robustness against the change in the DFR.

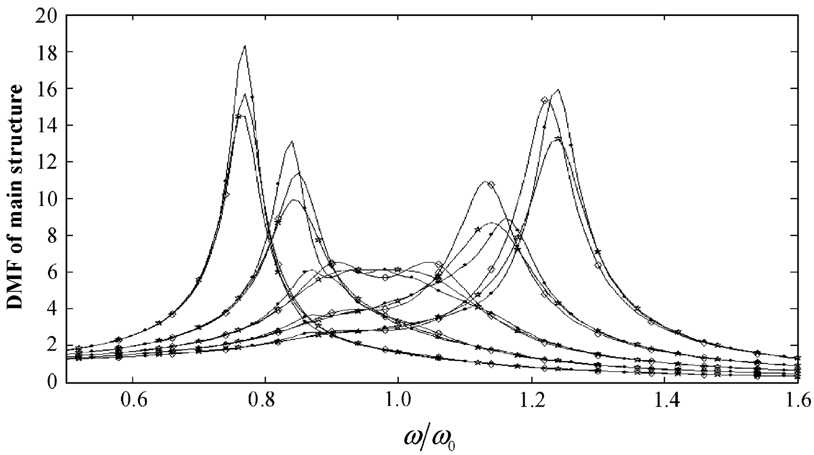

Fig. 13. Frequency response function (FRF) of the main structure with the DTMD using the second type of optimum goal functions (namely the novel optimum goal function), but the TMD and MTMD utilizing the first type of optimum objective functions commonly used in the case of DFRV, $\Gamma = [0.8 \quad 0.9 \quad 1.0 \quad 1.1 \quad 1.2]$: DTMD (—☆—); TMD (—◇—); MTMD (—•—).

Table 6
The minimum max.DMF (min.max.DMF) of the main structure respectively with the DTMD, MTMD, and TMD in terms of the first type of optimum objective functions commonly used and second type of optimum objective functions (namely the novel optimum objective function) as well as modifying the weighting factors in the second type of optimum objective functions for both the DTMD and TMD

Types of the optimum goal functions used in the design	min.max.DMF of main structure with the DTMD	min.max.DMF of main structure with the MTMD	min.max.DMF of main structure with the TMD
Employing the first type of optimum objective functions	5.347	6.131	6.546
Employing the second type of optimum objective functions	6.112	7.285	6.668
Modifying the weighting factors in the second type of optimum objective functions for both the DTMD and TMD	6.996	7.285	7.208

(2) Employing the second type of optimum goal functions, the effectiveness of both the TMD and MTMD tends to diminish. But, both of them possess a significantly increasing robustness to the changes in the DFR. Although the DTMD is better than the MTMD in terms of the effectiveness, the former is worse than the latter in terms of the robustness to the changes in the DFR.
(3) Via further modifying the weighting factors in the second type of optimum objective functions, the DTMD and MTMD can approximately attain the same level of vibration suppression, especially the same level of robustness to the changes in the DFR. However, the TMD cannot closely match the MTMD in the effectiveness and robustness to the changes in the DFR through further modifying the weighting factors in the second type of optimum objective functions.

6. Comparison between the MTMD and DTMD in terms of the frequency band width coefficient (FBWC)

All the TMD units in the MTMD form a subordinate system. The total number of the TMD units equals that of the resonance natural frequencies, namely the tuned natural frequencies of this subordinate system.

Table 7
The frequency band width coefficient (FBWC) of both the DTMD and MTMD in terms of the first type of optimum objective functions commonly used and second type of optimum objective functions (namely the novel optimum objective function)

Types of the optimum goal functions used in the design	FBWC of the DTMD for $\mu = 0.01$	FBWC of the DTMD for $\mu = 0.03$	FBWC of the MTMD for $\mu = 0.01$	FBWC of the MTMD for $\mu = 0.03$
Employing the first type of optimum goal functions	0.165	0.279	0.129	0.228
Employing the second type of optimum goal functions	0.313	0.453	0.120	0.285
Modifying the weighting factors in the second type of optimum goal functions for the DTMD	0.256	0.404	0.120	0.285

Suppose the total number of the TMD units is taken as n, it is evident that the MTMD will possess n resonance frequencies, while the total number of the natural frequencies of this subordinate system also equals n. The resonance frequencies of this subordinate system distribute around the structural controlled natural frequency, thus covering this controlled natural frequency. When the input excitation frequency is close to that of the main structure, high level of vibration reduction will be attained by large-amplitude motion of the MTMD.

Further, the ratio of the outcome of subtracting the minimum from the maximum of the natural frequencies to the structural controlled natural frequency is referred herein as the FBWC. The FBWC reflects the width of input excitation frequencies in which the MTMD will render a desirable level of vibration suppression. For example, for the MTMD with the total number of the TMD units equal to five and with the total mass ratio being 0.03, the ratios of the resonance frequencies of subordinate system to the structural controlled natural frequency, respectively, equal 0.871, 0.920, 0.969, 1.018, 1.067, thus FBWC = 0.196. After the main structure, a linear constant system reaches the steady state of vibration, its natural frequency approximately matches that of the input excitation. The main structure will generate the driving force, whose natural frequency equals that of the input excitation, to vibrate the subordinate system attached on it. Given that the input frequency is close to those n resonance frequencies, the vibration amplitude of subordinate system is relatively large; on the contrary, that of the main structure is smaller. If the frequencies of the input excitation are within the FBWC of the MTMD, the subordinate system will in resonance then lead to high level of vibration suppression. The DTMD, in fact, being a two-order subordinate system consisting of one larger TMD and one smaller TMD, possesses two resonance natural frequencies, which could be obtained in light of the mass and stiffness matrices of the DTMD. These two resonance natural frequencies will cover the structural controlled natural frequency in that those two frequencies distribute around the structural controlled natural frequency. Similarly, the ratio of the outcome of subtracting the smaller frequency from the larger one to the controlled natural frequency of the structure is FBWC. For instant, with the total mass ratio equal to 0.03, the ratios of the resonance frequencies of two-order subordinate system to the structural controlled natural frequency are, respectively, 0.8399 and 1.1192, consequently FBWC = 0.2793. The DTMD can then render high level of vibration reduction in the case where the input excitation frequencies are within this FBWC. Evidently, besides good robustness, the DTMD possesses convenient maintenance as well as simplified configuration in comparison with the MTMD. Table 7 provides the FBWC of both the DTMD and MTMD with the total mass ratio respectively equal to 0.01 and 0.03. It is evident that the DTMD is larger than the MTMD in terms of the FBWC, thus demonstrating that the DTMD possesses significantly better robustness to the NFT than the MTMD.

7. Conclusion

The DTMD consists of one larger TMD and one smaller TMD. The vibration energy dissipation way of the DTMD is different from that of both the TMD and MTMD. The DTMD possesses better effectiveness and higher robustness to the changes in the DFR in comparison with the TMD. The DTMD designed using the second type of optimum objective functions, namely the novel optimum objective function proposed in this paper, practically provides the same effectiveness and robustness to the changes in the DFR as the MTMD with the total number of the TMD units equal to five and with equal total mass ratio (from Ref. [12]). Likewise, the DTMD possesses significantly higher robustness to the changes in the DFR than the TMD. On the other hand, the FBWC of the DTMD is larger than that of the MTMD with the total number of the TMD units equal to five with equal total mass ratio, thus manifesting that the DTMD possesses significantly higher robustness to the NFT than the MTMD. Furthermore, the numerical computation indicates that the optimum damping ratio of the larger TMD equals zero, thus no need of the dampers between the larger TMD in the DTMD and the structure, which implies that the DTMD possesses considerable convenient maintenance as well as simplified configuration in comparison with the MTMD. Therefore, the DTMD is an advanced control device with respect to both the TMD and MTMD.

Generally, the optimum damping ratio (ζ_2) of the smaller TMD in the DTMD is significantly larger than that of the traditional TMD. The vibration of the smaller TMD may thus be suppressed effectively. However, the physical principle of the DTMD is different from that of the TMD. The DTMD suppresses the vibration of structures through large relative motion between the larger and smaller TMDs to activate the dashpot of the smaller TMD with high damping ratio to dissipate the vibration energy, consequently attenuating this large relative motion. Another important feature of the DTMD is that no dashpot is to be required between the larger TMD and a structure; while the dashpot is required between the TMD and a structure. However, it is worth pointing out here that sometimes the dashpot between the TMD and a structure plays a negative role in suppressing the vibration of structures. For example, the dashpot accelerates the vibration of structures, when $\dot{x}_s(\dot{x}_s - \dot{x}_{\mathrm{TMD}}) \leqslant 0$, in which $\dot{x}_s$ and $\dot{x}_{\mathrm{TMD}}$ refer to the velocities of the main structure and TMD with respect to the ground, respectively.

The two types of optimum goal functions all include the variables (the tuning frequency ratios λ_1 and λ_2, damping ratios ζ_1 and ζ_2, and mass ratio μ_H) to be optimized, and thus the control performance of the DTMD is dependent on the mass ratio (μ_H). Evidently, the mass ratio (μ_H) is concerned with the total mass ratio (μ) selected in designing the DTMD. Generally, the total mass ratio is within the range from 0.01 to 0.03 in practical applications; consequently, the optimum mass ratio (μ_H) is within the range from 0.028 to 0.083 when using the first type of optimum goal functions. However, the optimum mass ratio (μ_H) obtained using the second type of optimum goal functions is significantly larger than that using the first type of optimum goal functions.

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